Effects of dissipation on a quantum critical point with disorder

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We study the effects of dissipation on a disordered quantum phase transition with O(N) order parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a non-perturbative infinite-randomness critical point with unconventional activated dynamical scaling while superohmic damping leads to conventional behavior. We discuss applications to the superconductor-metal transition in nanowires and to Hertz' theory of the itinerant antiferromagnetic transition.

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The low-temperature properties of quantum many-particle systems are often sensitive to small amounts of impurities or defects. Close to quantum phase transitions (QPTs), the interplay between quantum fluctuations and random fluctuations due to disorder can destabilize the conventional critical behavior, leading to exotic phenomena such as quantum Griffiths effects [1, 2] and infinite-randomness critical points [3] as well as smeared phase transitions [4] (for a recent review see, e.g., Ref. [5]).

In particular, the QPTs in disordered quantum Ising magnets are governed by infinite-randomness critical points [3, 6] which display slow *activated* dynamical scaling. In a dissipative environment, the dynamics becomes even slower. In the experimentally relevant case of Ohmic dissipation, the tunneling of sufficiently large droplets (the ones normally responsible for Griffiths phenomena) is completely suppressed [7, 8]. As a result, the sharp quantum phase transition is destroyed by smearing [4].

In contrast, in dissipationless systems with *continuous* O(N) order parameter symmetry, disorder does *not* induce exotic infinite-randomness behavior in dimensions $d > 1$ [9]. This changes in the presence of Ohmic dissipation. It was recently shown that large locally ordered droplets are not frozen (in contrast to the Ising case. Instead they display the exponentially slow dynamics associated with a quantum Griffiths phase [10]. This leads to the important question of whether the QPTs of continuous symmetry order parameters with Ohmic dissipation are also of infinite-randomness type.

In this Letter, we answer this question and elucidate the nature of the transition by applying a strong-disorder renormalization group (RG) to the Landau-Ginzburg-Wilson (LGW) order-parameter field theory of the problem. Our results are summarized as follows: The QPT is controlled by an exotic infinite-randomness fixed point in the universality class of the random transverse-field Ising model. The dynamical scaling is activated rather than power-law, i.e., correlation time τ and correlation length ξ are related via $\ln\tau \sim \xi^{\psi}$, with ψ the tunneling exponent. With decreasing temperature, the order parameter susceptibility diverges as $\chi \sim [\ln(1/T)]^{2\phi-d/\psi}/T$, and the specific heat vanishes as $C \sim [\ln(1/T)]^{-d/\psi}$.

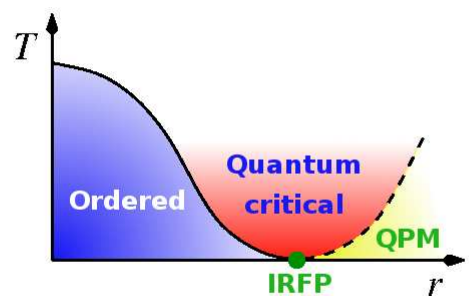

FIG. 1: (color online) Temperature–coupling phase diagram for Ohmic dissipation. IRFP denotes the infinite-randomness critical point. The phase boundary (solid) and the crossover line (dashed) between the quantum critical and quantum paramagnetic (QPM) regions take unusual exponential forms leading to a wide quantum critical region. Both phases contain Griffiths regions near the IRFP [10].

Here, ϕ is the cluster size exponent. Close to the QPT, the finite-temperature phase boundary takes the unusual form $T_c \sim \exp(-\mathrm{const}\,|r|^{-\nu\psi})$ with r the dimensionless distance from the QPT and ν the correlation length exponent. The exponents ψ, ϕ, and ν are universal and identical to those of the random transverse-field Ising model. The resulting phase diagram is shown in Fig. 1.

Our starting point is a quantum LGW free energy functional for an N-component ($N > 1$) order parameter φ in d dimensions. The clean action reads

$$S = \int \mathrm{d}y\mathrm{d}x\ \varphi(x)\Gamma(x,y)\varphi(y) + \frac{u}{2N}\int \mathrm{d}x\ \varphi^4(x)\,, \quad (1)$$

where $x \equiv (\mathbf{x}, \tau)$ comprises imaginary time τ and position $\mathbf{x}$, $\int \mathrm{d}x \equiv \int \mathrm{d}\mathbf{x} \int_0^{1/T} \mathrm{d}\tau$, and $\Gamma(x, y)$ is the bare inverse propagator (two-point vertex) whose Fourier transform reads $\Gamma(\mathbf{q}, \omega_n) = r + \xi_0^2\mathbf{q}^2 + \gamma\,|\omega_n|^{2/z_0}$ with r the bare distance from criticality (the bare gap). ξ_0 is a microscopic length scale, and ω_n is a Matsubara frequency. The damping coefficient γ depends on the coupling of the order parameter to the dissipative bath and the spectral density of the bath modes. We are mostly interested in overdamped (Ohmic) spin dynamics ($z_0 = 2$). However,

to demonstrate the special role of $z_0 = 2$, we also consider variable z_0. Quenched disorder can be introduced by making the distance from criticality r a random function of position $r \to r + \delta r(\mathbf{x})$. Analogously, disorder is introduced into ξ_0 and/or γ.

To apply the real-space based strong-disorder RG [11, 12], we discretize the action (1) by defining discrete coordinates $\mathbf{x}_j$ and rotor variables $\varphi_j(\tau)$. It is important to note that the rotors do *not* describe individual microscopic degrees of freedom, they rather represent the average order parameter in a volume ΔV large compared to ξ_0 but small compared to the correlation length ξ, i.e., $\varphi_j(\tau) = \int_{\Delta V} \mathrm{d}\mathbf{y}\, \varphi(\mathbf{x}_j + \mathbf{y})$.

We first consider the large-N limit of our action where all calculations can be carried out explicitly. We will later show that the results apply to all $N > 1$. In the large-N limit, the discrete action reads

$$\begin{aligned} S &= \frac{T}{E_0} \sum_i \sum_{\omega_n} \left(r_i + \lambda_i + \gamma_i \left|\omega_n\right|^{2/z_0} \right) \left|\phi_i(\omega_n)\right|^2 \\ &\quad - \frac{T}{E_0} \sum_{\langle i,j \rangle} \sum_{\omega_n} \phi_i(-\omega_n) J_{ij} \phi_j(\omega_n) \,, \end{aligned} \tag{2}$$

where r_i, $\gamma_i > 0$ and the nearest-neighbor interactions $J_{ij} > 0$ are random quantities, E_0 is a microscopic energy scale used to make the field dimensionless, and $\phi_j(\omega_n) = E_0 \int_0^{1/T} \varphi_j(\tau) e^{i\omega_n \tau} \mathrm{d}\tau$. The Lagrange multiplier λ_i enforces the large-N constraint $\langle (\varphi_i^{(k)}(\tau))^2 \rangle = 1$ for each order parameter component $\varphi_i^{(k)}$. The local distance from criticality, $\epsilon_i = r_i + \lambda_i$, contains all single-site renormalizations and is thus always positive.

The basic idea of the strong-disorder (Ma-Dasgupta-Hu) RG is to successively integrate out local high-energy degrees of freedom [3, 11, 12]. Here, the competing local energies are the local gaps ϵ_i and the interactions J_{ij}. In the bare theory (2), they are independent random variables with distributions $Q(\epsilon)$ and $P(J)$, respectively.

In each RG step, we choose the largest local energy $\Omega = \max\{\epsilon_i, J_{ij}\}$. If it is a gap, say ϵ_2, the unperturbed part of the action is $S_0 = (T/E_0) \sum_{\omega_n} (\epsilon_2 + \gamma_2 \left|\omega_n\right|^{2/z_0}) |\phi_2(\omega_n)|^2$. The coupling of ϕ_2 to the neighboring sites, $S_1 = -(T/E_0) \sum_{j,\omega_n} J_{2j} \phi_2(-\omega_n) \phi_j(\omega_n)$, is treated perturbatively. Keeping only the leading low-frequency terms that arise in 2nd order of the cumulant expansion, we obtain new interactions $\tilde{S} = -(T/E_0) \sum_{\omega_n} \phi_i(-\omega_n) \tilde{J}_{ij} \phi_j(\omega_n)$ between all sites that used to couple to ϕ_2, with

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2} J_{2j}}{\epsilon_2} \,. \tag{3}$$

At the end of the RG step, ϕ_2 is dropped from the action.

If the largest local energy is an interaction, say J_{23}, we solve the two-site cluster $S_0 = (T/E_0) \sum_{\omega_n} \sum_{i=2,3} (\epsilon_i + \gamma_i \left|\omega_n\right|^{2/z_0}) |\phi_i(\omega_n)|^2 - (T/E_0) \sum_{\omega_n} J_{23} \phi_2(-\omega_n) \phi_3(\omega_n)$ exactly while treating the interactions with all other sites as perturbations. The calculation is straightforward but lengthy; details will be published elsewhere. For $J_{23} \gg \epsilon_2, \epsilon_3$, the two rotors ϕ_2 and ϕ_3 are essentially parallel; thus they can be replaced by a single rotor $\tilde{\phi_2}$ with effective renormalized action $\tilde{S} = (T/E_0) \sum_{\omega_n} (\tilde{\epsilon_2} + \tilde{\gamma_2} \left|\omega_n\right|^{2/z_0}) |\tilde{\phi_2}(\omega_n)|^2$. For Ohmic dissipation, $z_0 = 2$, the renormalized gap is given by

$$\tilde{\epsilon}_2 = 2 \frac{\epsilon_2 \epsilon_3}{J_{23}} \,, \tag{4}$$

implying the relation $\tilde{\gamma_2} = \gamma_2 + \gamma_3$ for the damping constants. The new rotor represents a cluster with effective moment (number of sites represented)

$$\tilde{\mu}_2 = \mu_2 + \mu_3 \,. \tag{5}$$

The renormalized interactions of the new rotor with each of the remaining ones are given by

$$\tilde{J}_{2j} = J_{2j} + J_{3j} \,. \tag{6}$$

The net result of the RG step is the elimination of one site and the reduction of the energy scale Ω together with renormalizations and reconnections of the lattice. Since the structure of the RG recursion relations (3-6) is identical to that of the random transverse-field Ising model [3, 6, 13], we conclude that our system belongs to the same universality class.

In $d = 1$, the RG step does not change the lattice topology. One can thus derive flow equations for the individual probability distributions of ϵ and J and solve them analytically [3]. In $d > 1$, new couplings are generated in each RG step, and an analytical solution is impossible. However, by implementing the recursion relations (3-6) numerically, Motrunich et al. [6] showed that there is a fixed point in the full joint distribution of the ϵ and J that corresponds to the critical point of the system. In both cases, the critical point is of infinite-randomness type. At criticality, the distribution of the ϵ_i and J_{ij} becomes singular and broadens without limit as $\Omega \to 0$ under renormalization which also provides an *a posteriori* justification for using the perturbative RG recursion relations (3-6). One may be concerned about the initial stages of the RG in a weakly disordered system, where the strong-disorder method is not very accurate. However, perturbative RG studies [14, 15] showed that there is *no* stable weak-disorder fixed point; instead the perturbative RG shows runaway flow towards large disorder.

We thus conclude that the infinite-randomness fixed point found here is *universal* and controls the transition for all nonzero disorder strength. Its critical behavior is characterized by three exponents ψ, ϕ, and ν. The tunneling exponent ψ controls the dynamical scaling, i.e., the relation between length scale L and energy scale Ω, which is of activated rather than power-law type

$$\ln(1/\Omega) \sim L^{\psi} \,. \tag{7}$$

It also controls the density n_Ω of surviving clusters via $n_\Omega \sim [\ln(1/\Omega)]^{-d/\psi}$. ϕ describes how the typical moment μ of a surviving cluster depends on Ω,

$$\mu \sim \ln^{\phi}(1/\Omega) \ , \tag{8}$$

while ν determines how the correlation length ξ depends on the distance r from criticality via $\xi \sim |r|^{-\nu}$. In one space dimension, the exponents are known exactly from Fisher's analytical solution [3]: $\psi = 1/2$, $\phi = (1+\sqrt{5})/2$ and $\nu = 2$. In two dimensions, they were determined numerically [6, 16], yielding $\psi = 0.42\ldots0.6$, $\phi = 1.7\ldots2.5$ and $\nu = 1.07\ldots1.25$. For $d = 3$, the scaling towards an infinite-randomness fixed point has been confirmed [6], but estimates of the exponent values are still lacking.

We emphasize the particular role played by the Ohmic dissipation ($z_0 = 2$) of the magnetic modes. To this end, we consider how the recursion relation (4) is modified for $z_0 \neq 2$. For the superohmic case, $z_0 < 2$, we find

$$\tilde{\epsilon_2}^{-x} = \alpha\left[\epsilon_2^{-x} + \epsilon_3^{-x}\right] + \mathcal{O}\left(J_{23}^{-x}\right) \ , \tag{9}$$

where $x = (2 - z_0)/z_0$ and α is a constant [17]. Thus, the multiplicative structure of (4) is replaced by an additive one. As a result, the local gaps ϵ are only weakly renormalized for $z_0 < 2$. Near criticality, the distribution of the interactions J becomes extremely singular while that of the gaps ϵ remains narrow. The critical point is therefore not of infinite-randomness type but conventional with power-law scaling $\tau \sim \xi^z$, although the dynamical exponent z can become arbitrarily large as $z_0 \to 2^-$. (Similar behavior was found at a percolation QPT [18].) For the subohmic case, $z_0 > 2$, the sharp transition is destroyed by smearing because rare regions can statically order independently from each other [5, 10].

We now turn to the behavior of observables which is similar to the random transverse-field Ising model [3, 6]. However, there are a few differences caused by the order parameter symmetry and the damping of the modes. Summing over all surviving clusters using (7) and (8) gives unusual scaling forms for the order parameter susceptibility χ and the specific heat,

$$\chi(r,T) = \frac{1}{T}\,[\ln(1/T)]^{2\phi-d/\psi}\,\Theta_\chi\left(r^{\nu\psi}\ln(1/T)\right) \ , \tag{10}$$

$$C(r,T) = \quad [\ln(1/T)]^{-d/\psi}\,\Theta_C\left(r^{\nu\psi}\ln(1/T)\right) \ , \tag{11}$$

where Θ_χ and Θ_C are universal scaling functions. At criticality, this leads to $C \sim [\ln(1/T)]^{-d/\psi}$ and $\chi \sim [\ln(1/T)]^{2\phi-d/\psi}/T$. The dynamic order parameter susceptibility at criticality can be derived similarly. On the real frequency axis, $\Im\chi(\omega + i0) = [\ln(1/\omega)]^{2\phi-d/\psi}/\omega$. This implies that low-temperature inelastic scattering experiments at the location of the order parameter Bragg peak should see a sharp upturn in the scattering intensity $\sim [\ln(1/\omega)]^{2\phi-d/\psi}/\omega$ with $\omega \to 0$. The scaling form (10) of the susceptibility can also be used to infer the shape of the phase boundary close to the QPT. The finite-temperature transition corresponds to a singularity in $\Theta_\chi(x)$ at some nonzero argument x_c. This yields the unusual form $T_c \sim \exp(-\mathrm{const}\,|r|^{-\nu\psi})$ shown in Fig. 1. The crossover line between the quantum critical and quantum paramagnetic regions displays analogous behavior.

The infinite randomness at criticality leads to peculiar behavior of the correlation functions. The *average* correlation function $\overline{G}(\mathbf{x})$ is dominated by the rare events of two distant sites belonging to the same surviving cluster. This yields [3, 6] $\overline{G}\,(\mathbf{x}) \sim |\mathbf{x}|^{-2(d-\phi\psi)}$. In contrast, a typical pair of sites is not in the same cluster, and develops exponentially weak correlations, $-\ln G_{\rm typ}\,(\mathbf{x}) \sim |\mathbf{x}|^{\psi}$.

Our explicit calculations are for the large-N limit of the O(N) LGW theory. To discuss their relevance for finite N, we contrast the cases of Ising ($N = 1$) and continuous ($N > 1$) symmetries. In the former, sufficiently strong Ohmic dissipation freezes the magnetic droplets (the localization transition in the dissipative two-state system [19]) leading to a destruction of the sharp transition by smearing [4]. Recently, this was confirmed in a numerical strong-disorder RG [20]. In contrast, for $N > 1$, isolated droplets continue to fluctuate but with a tunneling rate (gap) that is exponentially small in their size [10] because O(N) clusters are right *at* the lower critical dimension of the transition. This exponential size dependence of the gap requires the multiplicative structure of the recursion (4) for the merging of two rotors. We conclude that this multiplicative structure is valid for *all* $N > 1$. Since (3) just reflects standard perturbation theory, all our RG recursion relations and the resulting infinite-randomness critical point apply to the general O(N) case with $N > 1$. This has been confirmed for undamped dynamics ($z_0 = 1$) in the case of O(2) symmetry and for a general O(N) rotor model in Refs. [17].

In the remaining paragraphs, we summarize our results, we discuss applications, and we consider open questions. We have studied the effects of dissipation on the quantum phase transition in a quenched disordered system with O(N) symmetric order parameter. For Ohmic dissipation, we have found an infinite-randomness fixed point while the behavior for the superohmic case (including undamped dynamics) is conventional. For subohmic dissipation, the quantum phase transition is destroyed by smearing. This must be contrasted with the case of Ising symmetry for which undamped dynamics already leads to an infinite randomness fixed point [3, 6] while Ohmic dissipation causes a smeared transition [4]. All of these results are in agreement with a general classification [5] of phase transitions in the presence of weak disorder based on the effective dimensionality of rare regions: If finite-size regions are exactly at the lower critical dimension, the critical point is of infinite-randomness type. If they are below the lower critical dimension, the behavior is conventional; and if they can order (freeze) independently, the transition is smeared.

Our theory has several applications. For instance, the superconductor-metal quantum phase transition observed in thin nanowires [21] was studied using a LGW theory analogous to (1) in one dimension with an O(2) (complex) order parameter and Ohmic dissipation [22]. The effects of disorder on the thermodynamics of this problem are described by our theory. Transport properties can also be calculated using the methods of Refs. [23]. Analogously, our theory should apply to arrays of resistively shunted Josephson junctions.

A second potential application is the Hertz-Millis theory [24, 25] of the (incommensurate) itinerant antiferromagnetic transition. In this theory, a LGW free energy analogous to (1) is derived from a microscopic electron Hamiltonian $H = H_{\rm band} + H_{\rm int} + H_{\rm dis}$ consisting of a nontrivial band structure $H_{\rm band}$, a Hubbard-like interaction $H_{\rm int}$ and a random potential $H_{\rm dis}$ by integrating out the fermionic degrees of freedom in favor of the bosonic order-parameter field. The validity of the Hertz-Millis approach to these transitions is still a controversial question as several experiments, in particular in heavy fermion materials [26], have shown marked differences from the predicted behavior. Disorder effects are a much-discussed possible reason for these discrepancies [26, 27]. Our theory provides explicit results on how dissipation and disorder can yield activated dynamics, quantum Griffiths phenomena, and non-Fermi liquid behavior. This should make an experimental verification or falsification of the disorder scenario much easier. Note that attention must be paid to the long-range RKKY part of the interaction neglected in (1). It can produce an extra subohmic dissipation of locally ordered clusters [28] which leads to freezing into a "cluster glass" phase [29] at a low non-universal temperature $T_{\rm CG}$ determined by the strength of the subleading RKKY interactions. However, the infinite-randomness fixed point controls observables in the broad quantum critical region above.

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